Choosing White Balls

HackerRank

There are n balls in a row, and each ball is either *black* (`B`) or *white* (`W`). Perform k removal operations with the goal of *maximizing the number of white balls* picked. For each operation i (where $1 \leq i \leq k$):

1. Choose an integer, x_i, uniformly and independently from 1 to $n - i + 1$ (inclusive).
2. Remove the ${x_i}^{th}$ ball from either the left end or right end of the row, which decrements the number of available balls in the row by 1. You can choose to remove the ball from whichever end in each step maximizing the expected total number of white balls picked at the end.

Given a string describing the initial row of balls as a sequence of n `W`'s and `B`'s, find and print the expected number of *white* balls providing that you make all choices optimally. A correct answer has an *absolute* error of *at most* 10^{-6}.

Input Format

The first line contains two space-separated integers describing the respective values of n (the number of balls) and k (the number of operations).
The second line describes the initial sequence balls as a single string of n characters; each character is either `B` or `W` and describes a *black* or *white* ball, respectively.

Constraints

- $1 \leq k \leq n < 30$

Output Format

Print a single floating-point number denoting the expected number of *white* balls picked. Your answer is considered to be correct if it has an *absolute* error of *at most* 10^{-6}.

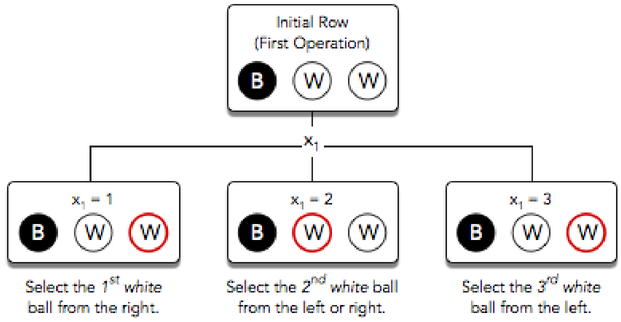
Sample Input 0

```
3 1
BWW
```

Sample Output 0

```
1.0000000000
```

Explanation 0



Independent of your choice of x, one *white* ball will always be picked so the expected number of *white* balls chosen after $k = 1$ operation is 1. Thus, we print 1 as our answer.

Sample Input 1

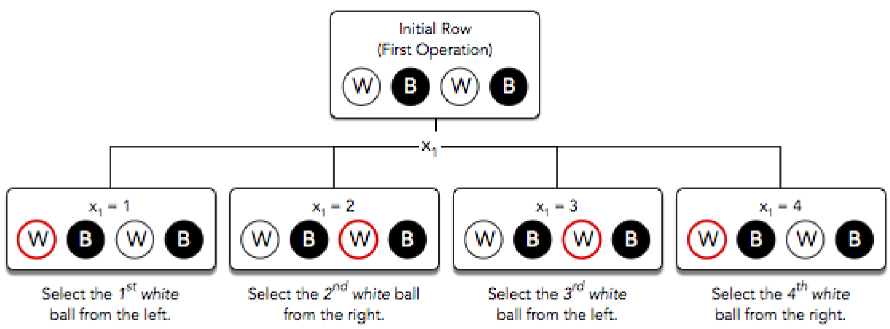
```
4 2
WBWB
```

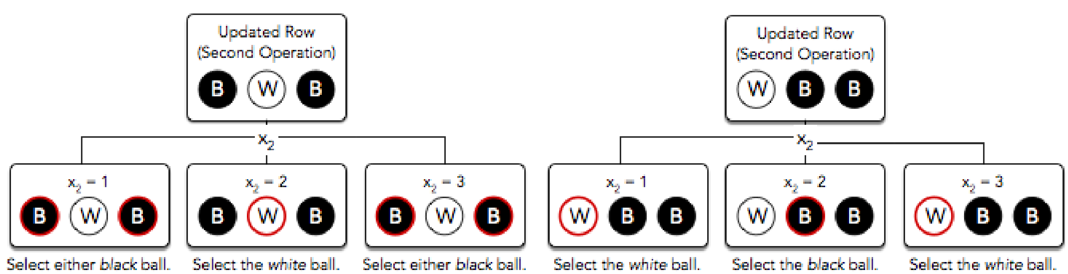
Sample Output 1

```
1.5000000000
```

Explanation 1

We perform the following $k = 2$ operations:

1. 

Independent of your choice of x, a *white* ball will always be chosen during the first operation (meaning the expected number of *white* balls in the first operation is 1).
2. 

For the second operation, there are 2 possible row orderings (depending on which ball was picked during the first operation). In the first possible row ordering, the probability of picking a *white* ball is $\frac{1}{3}$. In the second possible row ordering, the probability of picking a *white* ball is $\frac{2}{3}$. This means the expected number of *white* balls chosen in the second operation is $\frac{1}{2} \cdot \frac{1}{3} + \frac{1}{2} \cdot \frac{2}{3} = \frac{1}{2}$.

After performing all $k = 2$ operations, we print the total expected number of *white* balls chosen, which is $1 + \frac{1}{2} = 1.5$.