Divisor Exploration II

HackerRank

You are given q queries where each query is in the form of two integers, m and a, such that:

$$n = \prod_{i=1}^{m} p_i^{a+i}, \text{ where } p_i \text{ is the } i^{th} \text{ prime.}$$

For each query, find the following value:

$$result = \sum_{x|n} \sigma_1(x)$$

where $x|n$ denotes that each x is a divisor of n and $\sigma_1(x)$ is the *sum of the divisors* of x. Then print the value of $result \bmod (10^9 + 7)$ on a new line.

Input Format

The first line contains an integer, q, denoting the number of queries.
Each line i of the q subsequent lines contains two space-separated integers describing the respective values of m_i and a_i for query i.

Constraints

- $1 \leq q \leq 50$
- $1 \leq m \leq 10^5$
- $0 \leq a \leq 10^5$

Output Format

For each query, print a single integer denoting the value of $result \bmod (10^9 + 7)$ on a new line.

Sample Input 0

```
3
2 0
3 0
2 4
```

Sample Output 0

```
72
13968
196320
```

Explanation 0

For the first query, we are given $m = 2$ and $a = 0$. Recall that the sequence of prime numbers is $p = \{2, 3, 5, 7, 11, 13, \ldots\}$. We use $p_1 = 2$ and $p_2 = 3$ to calculate

$n = p_1^{a+1} \times p_2^{a+2} = 2^{0+1} \times 3^{0+2} = 18.$

The divisors of $n = 18$ are $\{1, 2, 3, 6, 9, 18\}$. We then use them to calculate the following:

$$\begin{aligned} result &= \sigma_1(1) + \sigma_1(2) + \sigma_1(3) + \sigma_1(6) + \sigma_1(9) + \sigma_1(18) \\ &\Rightarrow 1 + (1+2) + (1+3) + (1+2+3+6) + (1+3+9) + (1+2+3+6+9+18) \\ &\Rightarrow 1 + 3 + 4 + 12 + 13 + 39 \\ &\Rightarrow 72 \end{aligned}$$

Finally, we print the value of $result \bmod (10^9 + 7) = 72 \bmod (10^9 + 7) = 72$ on a new line. We then follow the same process to answer the second and third queries.