Mutual Recurrences

HackerRank

Since you know how to compute large Fibonacci numbers quickly using *matrix exponentiation*, let's take things to the next level.

Let a, b, c, d, e, f, g and h be positive integers. We define two bi-infinite sequences

$$(\ldots, x_{-2}, x_{-1}, x_0, x_1, x_2, \ldots)$$

and

$$(\ldots, y_{-2}, y_{-1}, y_0, y_1, y_2, \ldots)$$

as follows:

$$x_n = \begin{cases} x_{n-a} + y_{n-b} + y_{n-c} + n \cdot d^n & \text{if } n \geq 0 \\ 1 & \text{if } n < 0 \end{cases}$$

and

$$y_n = \begin{cases} y_{n-e} + x_{n-f} + x_{n-g} + n \cdot h^n & \text{if } n \geq 0 \\ 1 & \text{if } n < 0 \end{cases}$$

Given n and the eight integers above, find x_n and y_n. Since these values can be very large, output them modulo 10^9.

This link may help you get started: http://fusharblog.com/solving-linear-recurrence-for-programming-contest/

Input Format

The first line of input contains T, the number of test cases.
Each test case consists of a single line containing nine space separated integers: a, b, c, d, e, f, g, h and n, respectively.

Constraints
$1 \leq T \leq 100$
$1 \leq a, b, c, d, e, f, g, h < 10$
$1 \leq n \leq 10^{18}$

Output Format

For each test case, output a single line containing two space separated integers, $x_n \bmod 10^9$ and $y_n \bmod 10^9$.

Sample Input

```
3
1 2 3 1 1 2 3 1 10
1 2 3 2 2 1 1 4 10
1 2 3 4 5 6 7 8 90
```

Sample Output

```
1910 1910
909323 11461521
108676813 414467031
```

Explanation

In the second test case, the following is a table of values x_i and y_i for $0 \le i \le 10$:

i	x_i	y_i
0	3	3
1	7	11
2	19	49
3	57	241
4	181	1187
5	631	5723
6	2443	27025
7	10249	125297
8	45045	571811
9	201975	2574683
10	909323	11461521

Remember that $x_i = y_i = 1$ if $i < 0$.

One can verify this table by using the definition above. For example:

$$\begin{aligned} x_5 &= x_{5-1} + y_{5-2} + y_{5-3} + 5 \cdot 2^5 \\ &= x_4 + y_3 + y_2 + 160 \\ &= 181 + 241 + 49 + 160 \\ &= 631 \\ y_5 &= y_{5-2} + x_{5-1} + x_{5-1} + 5 \cdot 4^5 \\ &= y_3 + x_4 + x_4 + 5120 \\ &= 241 + 181 + 181 + 5120 \\ &= 5723 \\ x_2 &= x_{2-1} + y_{2-2} + y_{2-3} + 2 \cdot 2^2 \\ &= x_1 + y_0 + y_{-1} + 8 \\ &= 7 + 3 + 1 + 8 \\ &= 19 \end{aligned}$$