

Subset Component

You are given an array with n 64-bit integers: $d[0], d[1], \dots, d[n - 1]$.

$\text{BIT}(x, i) = (x \gg i) \& 1$, where $B(x, i)$ is the i^{th} lower bit of x in binary form. If we regard every bit as a vertex of a graph G , there is an undirected edge between vertices i and j if there is a value k such that $\text{BIT}(d[k], i) == 1 \ \&\& \ \text{BIT}(d[k], j) == 1$.

For every subset of the input array, how many **connected-components** are there in that graph?

A connected component in a graph is a set of nodes which are accessible to each other via a path of edges. There may be multiple connected components in a graph.

Example

$d = \{1, 2, 3, 5\}$

In the real challenge, there will be 64 nodes associated with each integer in d represented as a 64 bit binary value. For clarity, only 4 bits will be shown in the example but all 64 will be considered in the calculations.

| Decimal | Binary | Edges | Node ends |
|------------|--------|-------|-----------|
| $d[0] = 1$ | 0001 | 0 | |
| $d[1] = 2$ | 0010 | 0 | |
| $d[2] = 3$ | 0011 | 1 | 0 and 1 |
| $d[3] = 5$ | 0101 | 1 | 0 and 2 |

Consider all subsets:

| Subset | Edges | | Connected components |
|--------------|-------|-------|----------------------|
| | Count | Nodes | |
| {1} | 0 | | 64 |
| {2} | 0 | | 64 |
| {3} | 1 | 0-1 | 63 |
| {5} | 1 | 0-2 | 63 |
| {1, 2} | 0 | | 64 |
| {1, 3} | 1 | 0-1 | 63 |
| {1, 5} | 1 | 0-2 | 63 |
| {2, 3} | 1 | 0-1 | 63 |
| {2, 5} | 1 | 0-2 | 63 |
| {3, 5} | 2 | 0-1-2 | 62 |
| {1, 2, 3} | 1 | 0-1 | 63 |
| {1, 2, 5} | 1 | 0-2 | 63 |
| {1, 3, 5} | 2 | 0-1-2 | 62 |
| {2, 3, 5} | 2 | 0-1-2 | 62 |
| {1, 2, 3, 5} | 2 | 0-1-2 | 62 |
| Sum | | | 944 |

The values 3 and 5 have 2 bits set, so they have 1 edge each. If a subset contains only a 3 or 5, there will be one connected component with 2 nodes, and 62 components with 1 node for a total of 63.

If both 3 and 5 are in a subset, 1 component with nodes 0, 1 and 2 is formed since node 0 is one end of each edge described. The other 61 nodes are solitary, so there are 62 connected components total.

All other values have only 1 bit set, so they have no edges. They have 64 components with 1 node each.

Function Description

Complete the *findConnectedComponents* function in the editor below.

findConnectedComponents has the following parameters:

- *int d[n]*: an array of integers

Returns

- *int*: the sum of the number of connected components for all subsets of *d*

Input Format

The first row contains the integer *n*, the size of *d*[].

The next row has *n* space-separated integers, *d*[*i*].

Constraints

$$1 \leq n \leq 20$$

$$0 \leq d[i] \leq 2^{63} - 1$$

Sample Input 0

```
3
2 5 9
```

Sample Output 0

```
504
```

Explanation 0

There are 8 subset of {2, 5, 9}.

{}

=> We don't have any number in this subset => no edge in the graph => Every node is a component by itself => Number of connected-components = 64.

{2}

=> The Binary Representation of 2 is 00000010. There is a bit at only one position. => So there is no edge in the graph, every node is a connected-component by itself => Number of connected-components = 64.

{5}

=> The Binary Representation of 5 is 00000101. There is a bit at the 0th and 2nd position. => So there is an edge: (0, 2) in the graph => There is one component with a pair of nodes (0,2) in the graph. Apart from that, all remaining 62 vertices are independent components of one node each (1,3,4,5,6...63) => Number of connected-components = 63.

{9}

=> The Binary Representation of 9 is 00001001. => There is a 1-bit at the 0th and 3rd position in this

binary representation. => edge: (0, 3) in the graph => Number of components = 63

{2, 5}

=> This will contain the edge (0, 2) in the graph which will form one component

=> Other nodes are all independent components

=> Number of connected-component = 63

{2, 9}

=> This has edge (0,3) in the graph

=> Similar to examples above, this has 63 connected components

{5, 9}

=> This has edges (0, 2) and (0, 3) in the graph

=> Similar to examples above, this has 62 connected components

{2, 5, 9}

=> This has edges(0, 2) (0, 3) in the graph. All three vertices (0,2,3) make one component => Other 61 vertices are all independent components

=> Number of connected-components = 62

$$S = 64 + 64 + 63 + 63 + 63 + 63 + 62 + 62 = 504$$