

# Project Euler #122: Efficient exponentiation

This problem is a programming version of [Problem 122](#) from [projecteuler.net](#)

The most naive way of computing  $n^{15}$  requires fourteen multiplications:

$$n \times n \times \cdots \times n = n^{15}$$

But using a "binary" method you can compute it in six multiplications:

$$\begin{aligned} n \times n &= n^2 \\ n^2 \times n^2 &= n^4 \\ n^4 \times n^4 &= n^8 \\ n^8 \times n^4 &= n^{12} \\ n^{12} \times n^2 &= n^{14} \\ n^{14} \times n &= n^{15} \end{aligned}$$

However it is yet possible to compute it in only five multiplications:

$$\begin{aligned} n \times n &= n^2 \\ n^2 \times n &= n^3 \\ n^3 \times n^3 &= n^6 \\ n^6 \times n^6 &= n^{12} \\ n^{12} \times n^3 &= n^{15} \end{aligned}$$

We shall define  $m(k)$  to be the minimum number of multiplications to compute  $n^k$ . For example  $m(15) = 5$ .

For a given  $k$ , compute  $m(k)$ , and also output the sequence of multiplications needed to compute  $n^k$ . See the sample output for more details.

## Input Format

The first line of input contains  $T$ , the number of test cases.

Each test case consists of a single line containing a single integer,  $k$ .

## Constraints

$$\begin{aligned} 1 &\leq T \leq 500 \\ 2 &\leq k \end{aligned}$$

Input file #1:  $k \leq 111$ .

Input file #2:  $k \leq 275$ .

## Output Format

For each test case, first output  $m(k)$  in a single line. Then output  $m(k)$  lines, each of the form  $n^a * n^b = n^c$ , where  $a$ ,  $b$  and  $c$  are natural numbers. You may also output  $n$  instead of  $n^1$ . Use the  $*$  (asterisk/star) symbol, not the letter  $x$  or something else.

The sequence of multiplications must be valid. Any valid sequence will be accepted.

### Sample Input

```
2
2
15
```

### Sample Output

```
1
n^1 * n^1 = n^2
5
n * n = n^2
n^2 * n = n^3
n^3 * n^3 = n^6
n^6 * n^6 = n^12
n^12 * n^3 = n^15
```

### Explanation

The second case,  $k = 15$ , is the example given in the problem statement.

The sample output illustrates that you can use  $n$  instead of  $n^1$ .