

Project Euler #191: Prize Strings

This problem is a programming version of [Problem 191](#) from [projecteuler.net](#)

A particular school offers cash rewards to children based on their score history.

During an *l*-day period, a string (scores) is formed, for each child, in the following way:

$$s_1|s_2|\dots|s_l$$

where $0 \leq s_i \leq c - 1$ is the score of the child at the i^{th} day.

If they get 1 for *m* consecutive days **or** 0 on more than *n* occasion(s) then they forfeit their prize.

For a given *l*, *n*, *m* and *c*, let's call strings for which the child gets his prize **prize strings**, and denote $f(l, n, m, c)$ the number of prize strings for these parameters.

For example, with *l* = 4 (4-day period), *n* = 1, *m* = 3 and *c* = 3, it can be verified that $f(4, 1, 3, 3) = 43$ and here are the different prize strings that can be formed:

2|2|2|2 2|2|2|1 2|2|2|0 2|2|1|2 2|2|1|1 2|2|1|0 2|2|0|2 2|2|0|1 2|1|2|2 2|1|2|1
2|1|2|0 2|1|1|2 2|1|1|0 2|1|0|2 2|1|0|1 2|0|2|2 2|0|2|1 2|0|1|2 2|0|1|1 1|2|2|2
1|2|2|1 1|2|2|0 1|2|1|2 1|2|1|1 1|2|1|0 1|2|0|2 1|2|0|1 1|1|2|2 1|1|2|1 1|1|2|0
1|1|0|2 1|1|0|1 1|0|2|2 1|0|2|1 1|0|1|2 1|0|1|1 0|2|2|2 0|2|2|1 0|2|1|2 0|2|1|1
0|1|2|2 0|1|2|1 0|1|1|2.

You are given *L*, *N*, *m* and *c*, what is $\sum_{l=1}^L \sum_{n=1}^N f(l, n, m, c) \bmod 10^9 + 7$.

Input Format

The only line of each test case contains exactly four integers separated by single spaces: *L*, *N*, *m* and *c*

Constraints

- $1 \leq N \times L \leq 10^7$
- $1 \leq m \leq L$
- $2 \leq c \leq 10^{18}$

Output Format

Print the answer modulo $10^9 + 7$

Sample Input

4 1 3 3

Sample Output

73

Explanation

$f(4, 1, 3, 3) = 43$, $f(3, 1, 3, 3) = 19$, $f(2, 1, 3, 3) = 8$ and $f(1, 1, 3, 3) = 3$. Hence the sum is **73**.