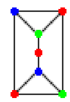
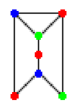
Project Euler #194: Coloured Configurations

This problem is a programming version of Problem 194 from projecteuler.net

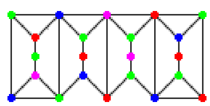
Consider graphs built with two graph units A and B, where the graph units are glued along their leftmost and rightmost vertical edges. For example, if graph units A and B are



and



then the graph



is built by gluing the units in the order $ABBA$.

A configuration of type (A, B, a, b, c) is a graph thus built of a units A and b units B, where the graph's vertices are coloured using up to c colours, so that no two adjacent vertices have the same colour.

The compound graph above is an example of a configuration of type $(A, B, 2, 2, 6)$, in fact of type $(A, B, 2, 2, c)$ for all $c \geq 4$.

Let $N(A, B, a, b, c)$ be the number of configurations of type (A, B, a, b, c), where two configurations of type (A, B, a, b, c) are considered the same if the corresponding coloured compound graphs are identical under translation transformation.

For example, with the graph units A and B as given above, $N(A, B, 1, 0, 3) = 24$, $N(A, B, 0, 2, 4) = 92928$ and $N(A, B, 2, 2, 3) = 20736$.

Find the number $N(A, B, a, b, c)$ modulo $10^9 + 7$ for a given pair of graph units A and B and the numbers a, b, and c.

Input Format

The first line contains three space-separated integers: a, b, and c, where a and b are number of graph units of type A and B respectively, and c is the number of colours.

The subsequent lines define the graph units A and B. Each graph is defined as follows:

1. The first line contains two space-separated integers: n which is the number of vertices and m which is the number of edges in the graph being defined.
2. Each line i of the n subsequent lines (where $1 \leq i \leq n$) contains two space-separated integers x_i and y_i describing the position of the i-th vertex in the graph being defined, with X as horizontal and

Y as vertical dimension.

3. Each line j of the m subsequent lines (where $1 \leq j \leq m$) contains two space-separated integers u_j and v_j describing the j-th edge as connecting vertices u_j and v_j.

The first two edges in the given lists for each graph are vertical edges of the same length for both graph units, the first edge being the leftmost edge, and the second one being the rightmost edge. These edges are used for the graph units binding.

Constraints

- $0 \leq a, b \leq 100$ as number of units A and B;
- $2 \leq c \leq 2000$ as number of colours;
- $0 \leq x_i, y_i \leq 1000$ for all i as coordinates of the i-th vertex, with no two vertices having the same (x, y) coordinates;
- $1 \leq u_j, v_j \leq n$ for all j as vertex indices of the j-th edge;
- $4 \leq n_A, n_B$
- $n_A + n_B \leq 64$, where n_A and n_B are number of vertices in the graph units A and B respectively.

  Both graph units A and B are simple connected planar graphs.

  The time restriction is a double of the usual time restriction.

Output Format

On a single line print one integer denoting the required number of graph configurations modulo $10^9 + 7$.

Sample Input 0

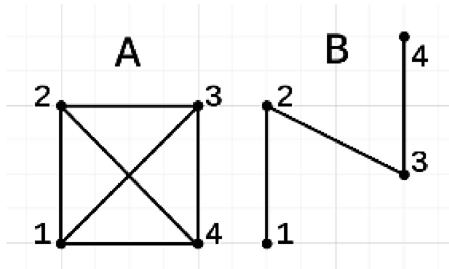
```
1 2 3
4 6
0 4
0 0
4 4
4 0
1 2
3 4
1 3
1 4
2 3
2 4
4 3
6 0
6 4
10 2
10 6
1 2
3 4
2 3
```

Sample Output 0

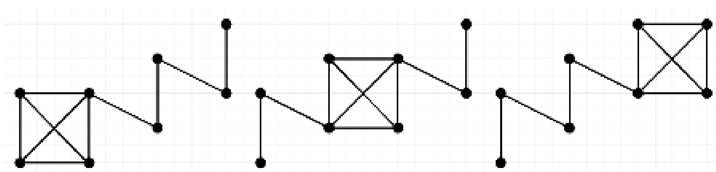
```
0
```

Explanation 0

The given graph units A and B are as follows:



There are only three ways to glue the graph units A and B for a configuration of type $(A, B, 1, 2, 3)$: ABB, BAB, and BBA.



However the graph unit A cannot be coloured with just 3 colours. Since the unit A should be included in any configuration of the type $(A, B, 1, 2, 3)$ then the total number of possible coloured configurations is zero.

Sample Input 1

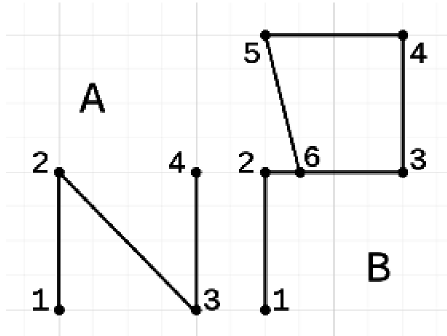
```
1 1 2
4 3
0 0
0 4
4 0
4 4
1 2
3 4
2 3
6 6
6 0
6 4
10 4
10 8
6 8
7 4
1 2
3 4
2 6
3 6
4 5
5 6
```

Sample Output 1

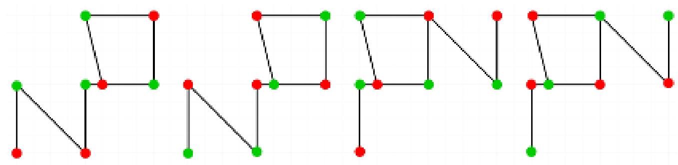
```
4
```

Explanation 1

The given graph units A and B are as follows:



There are two ways to glue the units A and B together: AB and BA; and there are two possible ways to colour each combined graph with 2 colours:



Therefore the number of configurations of type $(A, B, 1, 1, 2)$ is 4.

Sample Input 2

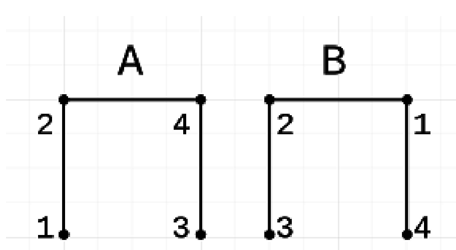
```
1 1 2
4 3
0 0
0 4
4 0
4 4
1 2
3 4
2 4
4 3
10 4
6 4
6 0
10 0
2 3
1 4
1 2
```

Sample Output 2

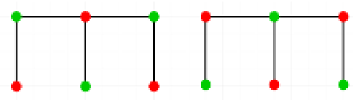
```
2
```

Explanation 2

The given graph units A and B are as follows:



The two possible bindings AB and BA generate the same compound graph that can be coloured in two ways:



Therefore the answer is 2.

Sample Input 3

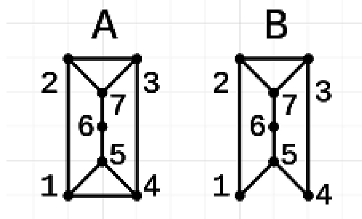
```
1 0 3
7 10
0 0
0 4
2 4
2 0
1 1
1 2
1 3
1 2
3 4
1 4
1 5
2 3
2 7
3 7
4 5
5 6
6 7
7 9
4 0
4 4
6 4
6 0
5 1
5 2
5 3
1 2
3 4
1 5
2 3
2 7
3 7
4 5
5 6
6 7
```

Sample Output 3

```
24
```

Explanation 3

The given graph units A and B are as follows:



Since b is zero, only the single unit of graph A is used for colouring. There are 24 ways to colour unit A with 3 colours: