Project Euler #226: A Scoop of Blancmange

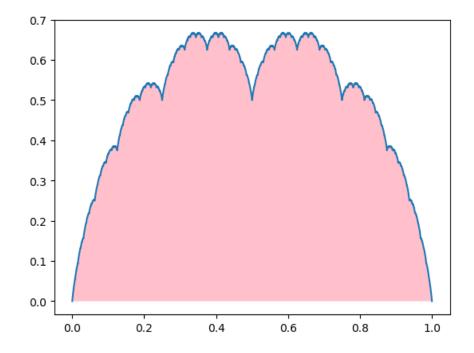
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This problem is a programming version of Problem 226 from projecteuler.net

For any real number x, define d(x) as the distance from x to its nearest integer. Let $r, s \ge 2$ be positive integers and consider the function $f_{r,s}$ defined on the real interval [0, 1] by:

$$f_{r,s}(x) = \sum_{n\geq 0} rac{d(r^n x)}{s^n}$$

For example, when r=s=2 we get the blancmange function shown bellow



Given a polynomial $P = \sum_{i=0}^m a_i X^i$, where a_i are integers. Let

$$I = \int_0^1 f_{r,s}(x) P(x) dx$$

It can be proved that I is a rational number, therefore we can write it as $I = \frac{p}{q}$ where p and q are integers. In addition, the constraints on the inputs guarantee that q is not divisible by the prime number 1004535809. In this case, find $p \cdot q^{-1}$ modulo 1004535809 (q^{-1} is the the inverse of q modulo 1004535809).

Input Format

The first line of each test file contains three space-separated integers r, s and m. The next line contains m+1 space-separated integers a_0, \ldots, a_m .

Constraints

- $2 \le r, s \le 10^9$.
- $0 \le m \le 2 \cdot 10^5$.
- $s \cdot r^i 1$ is not divisible by 1004535809 for all $0 \leq i \leq m+1$.
- $0 \leq a_i \leq 10^9$.
- $a_m > 0$.

Output Format

Print your answer in one line.

Sample Input 0

2 2 0 1

Sample Output 0

502267905

Explanation 0

The graph of $f_{2,2}$ is shown in the statement.

 $I = \int_0^1 f_{2,2}(x) dx = \frac{1}{2}$, hence $I = 1 \cdot 2^{-1} = 502267905 \mod 1004535809$.

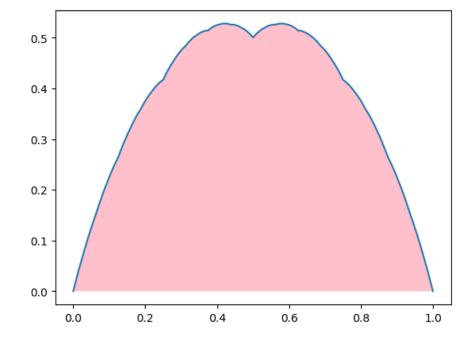
Sample Input 1

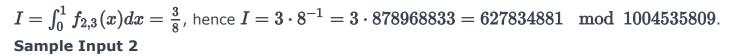
Sample Output 1

627834881

Explanation 1

Below is the graph of $f_{2,3}$





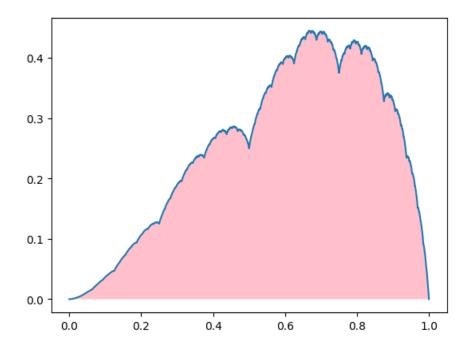
2 2 1 0 1

Sample Output 2

753401857

Explanation 2

The following is the graph of $x o x f_{2,2}(x)$



$I = \int_0^1 x f_{2,2}(x) dx = rac{1}{4}$, hence $I = 1 \cdot 4^{-1} = 753401857 \mod 1004535809$. Sample Input 3

42 57 5 490 480 625 34 405 968

Sample Output 3

617014829